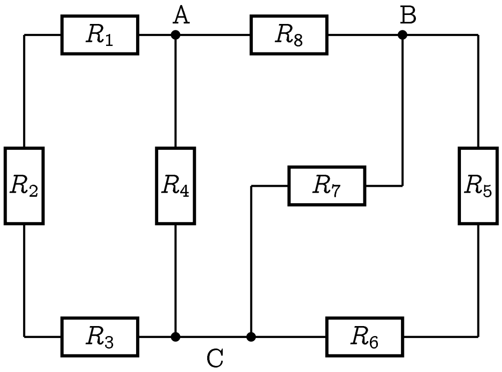
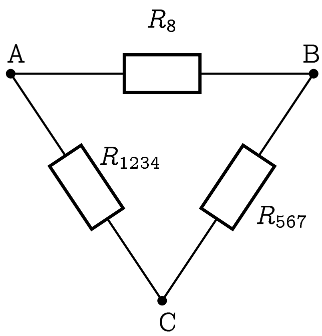
1 Übung

1.1



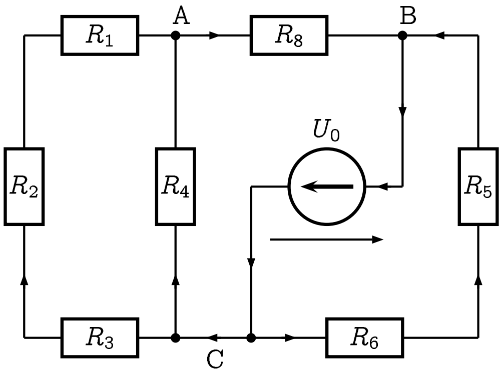
$$\begin{aligned}
R_1 &= 20\Omega \\
R_2 &= 300\Omega \\
R_3 &= 90\Omega \\
R_4 &= 10\Omega \\
R_5 &= 70\Omega \\
R_6 &= 800\Omega \\
R_7 &= 50\Omega \\
R_8 &= 200\Omega
\end{aligned}$$

$$\begin{aligned}
R_{1234} &= \frac{1}{\frac{1}{R_1+R_2+R_3}+\frac{1}{R_4}} &= \frac{1}{\frac{1}{20+300+90}+\frac{1}{10}}\Omega &\approx 9,762\Omega \\
R_{567} &= \frac{1}{\frac{1}{R_5+R_6}+\frac{1}{R_7}} &= \frac{1}{\frac{1}{800+70}+\frac{1}{50}}\Omega &\approx 47,283\Omega
\end{aligned}$$

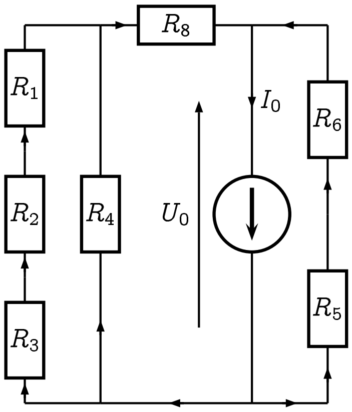


$$\begin{aligned}
R_{AB} &= \frac{1}{\frac{1}{R_8}+\frac{1}{R_{1234}+R_{567}}} &\approx 44,385\Omega \\
R_{BC} &= \frac{1}{\frac{1}{R_{567}}+\frac{1}{R_{1234}+R_8}} &\approx 38,585\Omega \\
R_{AC} &= \frac{1}{\frac{1}{R_{1234}}+\frac{1}{R_{567}+R_8}} &\approx 9,391\Omega
\end{aligned}$$

1.2



$$
\begin{aligned}
R_1 &= 20\Omega \\
R_2 &= 300\Omega \\
R_3 &= 90\Omega \\
R_4 &= 10\Omega \\
R_5 &= 70\Omega \\
R_6 &= 800\Omega \\
R_7 &\to U_0 = 10V \\
R_8 &= 200\Omega
\end{aligned}
$$

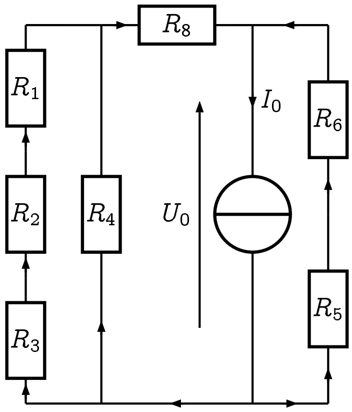


$$
\begin{aligned}
U_0 &= R_{ges} \cdot I_0 \\
R_{ges} &= \Big(R_8 + \big(R_4 \| (R_1 + R_2 + R_3)\big)\Big) \Big\| \Big(R_6 + R_5\Big) \\
I_0 &= \frac{U_0}{R_{ges}} \approx \frac{10V}{169,012\Omega} \approx 0.059A
\end{aligned}
$$

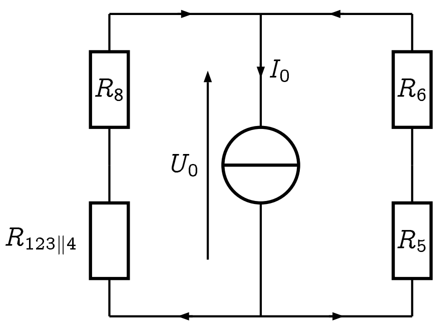
$$
\begin{aligned}
I_0 &= I_8 + I_{56} \qquad I_{56} = I_5 = I_6 \\
U_0 &= U_{56} = (R_5 + R_6) \cdot I_{56} \\
I_{56} &= \frac{U_0}{R_5 + R_6} \\
&= \frac{10V}{870\Omega} \approx 0,0115A \\
&\Rightarrow I_8 = I_0 - I_{56} \approx 0,0475A \\
\to & \qquad U_6 = R_6 \cdot I_6 \approx 0,805V
\end{aligned}
$$

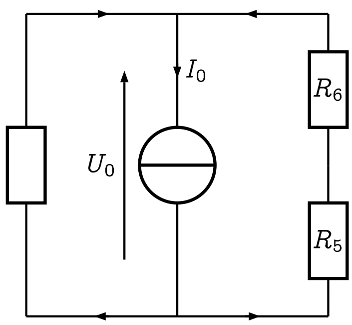
$$
\begin{aligned}
U_8 &= R_8 \cdot I_8 \\
U_0 &= U_8 + U_4 \qquad U_4 = U_{123} \\
U_{123} &= U_0 - R_8 \cdot I_8 \approx 0.5V \\
&\Rightarrow I_{123} = \frac{U_{123}}{R_1 + R_2 + R_3} \approx 0.0012A \\
& \quad (I_{123} = I_1 = I_2 = I_3) \\
\to & \qquad U_1 = R_1 \cdot I_1 \approx 0,024V \\
\to & \qquad U_2 = R_2 \cdot I_2 \approx 0,36V \\
\to & \qquad U_3 = R_3 \cdot I_3 \approx 0,108V
\end{aligned}
$$

1.3



$$
\begin{aligned}
R_1 &= 20\Omega \\
R_2 &= 300\Omega \\
R_3 &= 90\Omega \\
R_4 &= 10\Omega \\
R_5 &= 70\Omega \\
R_6 &= 800\Omega \\
R_7 &\to I_0 = 2A \\
R_8 &= 200\Omega
\end{aligned}
$$





$$
\begin{aligned}
U_0 &= R_{ges} \cdot I_0 \\
&\approx 169,012\Omega \cdot 2A \approx 338,024V
\end{aligned}
$$

$$
U_5 = U_0 \cdot \frac{R_5}{R_5 + R_6} \approx 27,2V
$$

$$
I_5 = \frac{U_5}{R_5} \approx 0,389A
$$

1.4

(a)

$$\begin{aligned} U_1 &= R_i \cdot I_i \\ &= 200\ k\Omega \cdot 100\ \mu A = 2 \cdot 10^3 \cdot 100 \cdot 10^{-6}\ V \\ &= 0,2V \end{aligned}$$

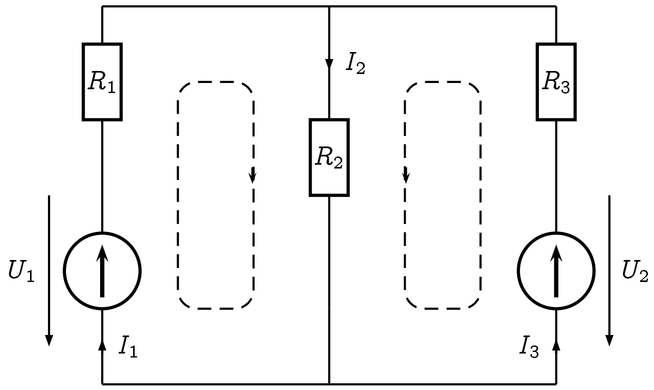
(b)

$$\begin{aligned} U_2 &= (R_i + R_1) \cdot I_i = U_1 + R_1 \cdot I_i \\ R_2 &= \frac{U_2 - U_1}{I_i} \\ &= \frac{1\ V - 0,2\ V}{100\ \mu A} = 8\ k\Omega \end{aligned}$$

$$\begin{aligned} U_3 &= (R_i + R_1 + R_2) \cdot I_i = U2 + R_3 \cdot I_i \\ R_3 &= \frac{U_3 - U_2}{I_i} \\ &= \frac{10\ V - 1\ V - 0,2\ V}{100\ \mu A} = 90\ k\Omega \end{aligned}$$

$$\begin{aligned} U_4 &= (R_i + R_1 + R_3 + R_3) \cdot I_i = U_3 + R_4 \cdot I_i \\ R_4 &= \frac{U_4 - U_3}{I_i} \\ &= \frac{100\ V - 10\ V}{100\ \mu A} = 900\ k\Omega \end{aligned}$$

1.5



$$
\begin{aligned}
-U_1 + R_1 I_1 + R_2 I_2 &= 0 \qquad (1)\\
-U_2 + R_3 I_3 + R_2 I_2 &= 0 \qquad (2)\\
I_1 + I_3 - I_2 &= 0 \qquad (3)
\end{aligned}
$$

(2) auflösen nach I_3:

$$I_3 = \frac{U_2 - R_2 I_2}{R_3}$$

einsetzen in (3):

$$I_1 + \frac{U_2 - R_2 I_2}{R_3} - I_2 = 0$$

auflösen nach I_2:

$$I_2 = \frac{I_1 R_3 + U_2}{R_2 + R_3}$$

einsetzen in (1):

$$-U_1 + R_1 I_1 + R_2 \frac{I_1 R_3 + U_2}{R_2 + R_3} = 0$$

auflösen nach I_1:

$$I_1 = \frac{-U_2 R_2 + U_1 (R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

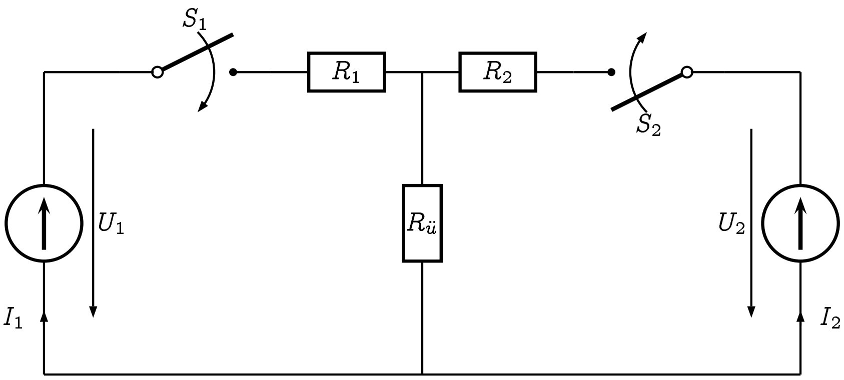
mit $(U_1 = 30V,\ U_2 = 5V,\ R_1 = 10\Omega,\ R_2 = 20\Omega,\ R_3 = 10\Omega)$ folgt:

$$I_1 = 1,6A; \qquad I_2 = 0,7A; \qquad I_3 = -0.9A$$

$$
\begin{aligned}
P_i &= U_i \cdot I_i\\
P_1 &= 30V \cdot 1,6A\\
&= 48W\\
P_2 &= 5V \cdot (-0,9)\\
&= -4,5W
\end{aligned}
$$

$$
\begin{aligned}
P_{R_i} &= I_{R_i}^2 \cdot R_i\\
P_{R_1} &= (1,6A)^2 \cdot 10\Omega\\
&= 25,6W\\
P_{R_2} &= (0,7A)^2 \cdot 20\Omega\\
&= 9,8W\\
P_{R_3} &= (-0,9A)^2 \cdot 10\Omega\\
&= 8,1W
\end{aligned}
$$

1.6



$$l = 2000m \quad \rho = 1,78 \cdot 10^{-2} \frac{\Omega\ mm^2}{m} \quad A = 1mm^2$$

a)

$$\begin{aligned} R_0 &= R_1 + R_2 = \rho\frac{x}{A} + \rho\frac{l-x}{A} = \rho\frac{l}{A} \\ &= 1,78 \cdot 10^{-2} \frac{\Omega\ mm^2}{m} \cdot 2 \cdot 10^3 \cdot \frac{1}{mm^2} \\ &= 35,6\Omega \end{aligned}$$

b) Y_1 (S_1 geschlossen, S_2 offen): $U_1 = 200V$, $I_1 = 2,5A$

$$U_1 = (R_1 + R_{ü})\ I_1 \qquad \rightarrow \quad R_1 + R_{ü} = \frac{200V}{2,5A} = 80\Omega$$

c) Y_2 (S_1 offen, S_2 geschlossen): $U_2 = 200V$, $I_2 = 2,94A$

$$U_2 = (R_2 + R_{ü})\ I_2 \qquad \rightarrow \quad R_2 + R_{ü} = \frac{200V}{2,94A} = 68,03\Omega$$

d)

$$
\begin{aligned}
R_1 + R_ü &= \rho\frac{x}{A} + R_ü = 80\Omega \\
R_ü &= 80\Omega - R_1 \\
\\
R_2 + R_ü &= \rho\frac{l-x}{A} + R_ü = 63,03\Omega \\
R_2 + 80\Omega - R_1 &= 63,03\Omega \\
R_2 - R_1 &= 63,03\Omega - 80\Omega = -11,97\Omega \\
\frac{\rho}{A}\left(l-2x\right) &= -11,97\Omega \\
-\frac{2\rho}{A}\,x &= -11,97\Omega - \frac{\rho\, l}{A} = -47,57\Omega \\
x &= 47,57\Omega \cdot \frac{A}{2\rho} \\
&= 47,57\Omega \cdot \frac{1mm^2\ m}{2\cdot 1,78\cdot 10^{-2}\Omega\ mm^2} \\
&= 1336,24m \\
\\
R_1 &= \rho\frac{x}{A} = 23,79\Omega \\
R_2 &= \rho\frac{l-x}{A} = 11,81\Omega \\
R_ü &= 56,21\Omega
\end{aligned}
$$

1.7

a) U_L: Spannungsteiler

$$R_L \| xR = \frac{xRR_L}{xR+R_L}$$

$$U_L = U_1 \cdot \frac{\frac{xRR_L}{xR+R_L}}{\left(1-x\right)R + \frac{xRR_L}{xR+R_L}} = U_1 \cdot \frac{xR_L}{x\left(1-x\right)R + R_L}$$

b) $U_1 = 10V; \quad R_L = 1k\Omega; \quad R = 2k\Omega; \quad x = 0,5$

$$P_R = P_{R_1} + P_{R_2} = I_1^2 \cdot \left(1-x\right)R + \frac{U_L^2}{xR}$$

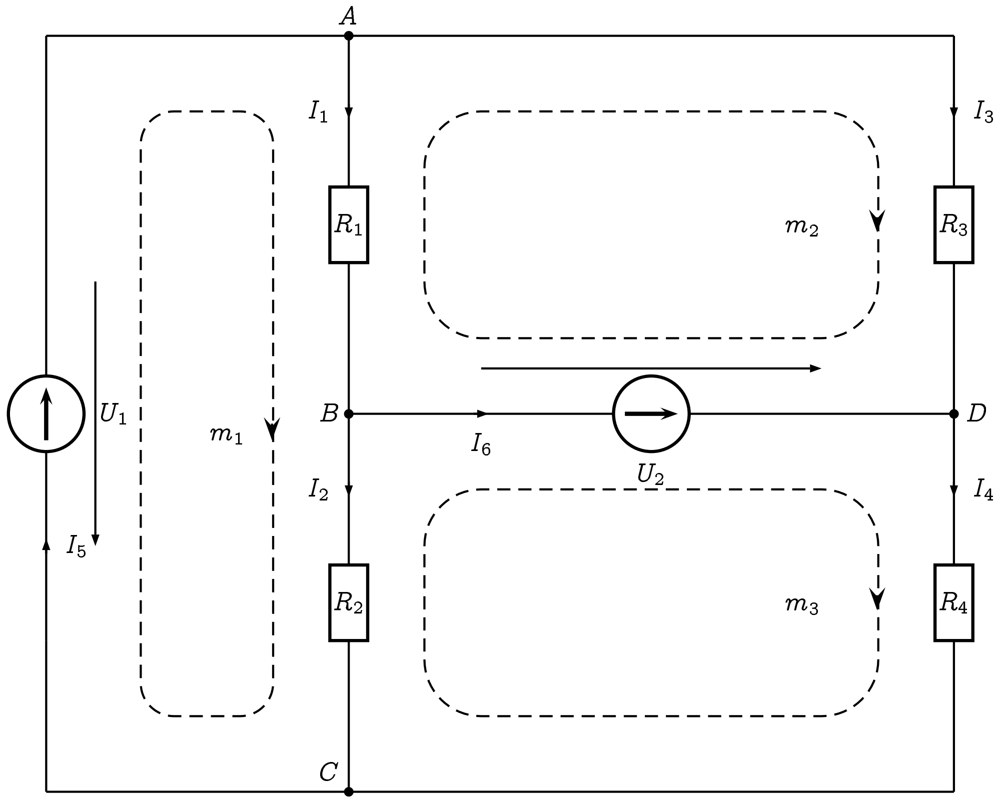
$$I_1 = \frac{U_1}{R_{ges}} \qquad \text{mit} \quad R_{ges} = \left(xR \| R_L\right) + \left(1-x\right)R = \frac{R\left(xR + R_L - x^2 R\right)}{xR+R_L}$$

$$R_{ges} = 1,5k\Omega,\ U_L = \frac{10}{3}V,\ I_1 = \frac{1}{150}A \quad \Rightarrow \quad P_R = \frac{1}{18}W \approx 55,5mW$$

c)

"Aufgrund der hohen Verlustleistung im Potentiometer"

1.8



Umlaufgleichungen:

$$
\begin{aligned}
m_1 &: & U_1 &= R_1\ I_1 + R_2\ I_2 \\
m_2 &: & U_2 &= -R_1\ I_1 + R_3\ I_3 \\
m_3 &: & -U_2 &= -R_2\ I_2 + R_4\ I_4
\end{aligned}
$$

Knotengleichungen:

$$
\begin{aligned}
A &: & I_5 &= I_1 + I_3 \\
B &: & I_1 &= I_6 + I_2 \\
C &: & I_5 &= I_2 + I_4 \\
D &: & I_4 &= I_6 + I_3
\end{aligned}
$$

Auflösen und Einsetzen liefert

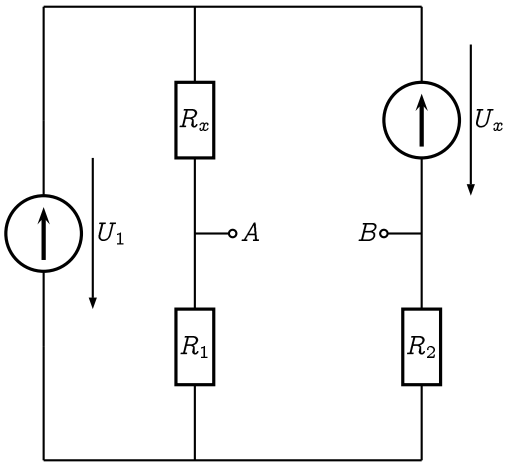
- I_1, I_2, I_3 aus Umlaufgleichungen
- I_4, I_5, I_6 aus Knotengleichungen

$$
\begin{aligned}
I_4 &= I_6 + I_3 \\
I_6 &= I_1 - I_2 \\
I_5 &= I_1 + I_3
\end{aligned}
$$

$$
\begin{aligned}
I_1 &= \frac{U_1 - R_2\ I_2}{R_1} \\
I_3 &= \frac{U_2 + U_1 - R_2\ I_2}{R_3} \\
I_2 &= \frac{U_2\ R_1\ R_3 + R_4\ U_1\ R_3 + R_4\ R_1\ U_2 + R_4\ R_1\ U_1}{R_2\ R_1\ R_3 + R_4\ R_2\ R_3 + R_4\ R_1\ R_3 + R_4\ R_2\ R_1}
\end{aligned}
$$

$$
\begin{aligned}
I_1 &= -\frac{10}{9}A \approx -1,11A \\
I_2 &= \frac{37}{18}A \approx 2,05A \\
I_3 &= \frac{37}{9}A \approx 4,11A \\
I_4 &= \frac{17}{18}A \approx 0,94A \\
I_5 &= 3A \\
I_6 &= -\frac{19}{6}A \approx -3,16A
\end{aligned}
$$

1.9



- $U_{AB} = 0$,
  gleiches potential an den Messpunkten
- jedoch kein Kurzschluss, $I_{AB} = 0$
- $U_{R_1} = U_{R_2}$
- $U_{R_1} = U_1 \cdot \frac{R_1}{R_1+R_x}$
- $U_x + U_{R_2} = U_1 \quad \rightarrow \quad U_x = U_1 - U_{R_2}$

a) $R_x = R_1 \quad \rightarrow \quad U_{R_1} = \frac{U_1}{2} = 5V \quad \Rightarrow \; U_x = 5V$

b) $U_x = 7V \quad \rightarrow \quad R_x = R_1 \left(\frac{U_1}{U_1 - U_x} - 1 \right) = \frac{70}{3} k\Omega \approx 23.333 k\Omega$

c) $U_1 = 12V,\; R_x = R_1 \quad \rightarrow \quad U_{R_1} = 6V \quad \Rightarrow \; U_x = 6V$

d) $U_1 = 12V,\; R_x = \frac{70}{3} k\Omega \quad \rightarrow \quad U_x = U_1 \left(1 - \frac{R_1}{R_1+R_x} \right) = 8.4V$

1.10

a)

$$R_i = R_3 \| (R_1 + R_2) = \frac{(R_1 + R_2) R_3}{R_1 + R_2 + R_3} = \frac{375}{40} \Omega = 9.375 \Omega$$

$$U_i = -I_0 R_1 \frac{R_3}{R_1 + R_2 + R_3} = -7.5V \quad \Rightarrow \quad I_i = \frac{U_i}{R_i} = -0.8A$$

b)

$$U_L = -I_0 R_1 \; \frac{\frac{R_3 R_L}{R_3 + R_L}}{R_1 + R_2 + \frac{R_3 R_L}{R_3 + R_L}} = \frac{-I_0 R_1 \; R_3 R_L}{(R_1 + R_2) \cdot (R_3 + R_L) + R_3 R_L} = -6V$$

$$P_L = U_L \cdot I_L \; = \frac{U_L^2}{R_L} = 0.96W$$

c)

$$I_{AB} = \frac{U_1}{R_u} \quad \Rightarrow \quad U_{AB} = (-I_0 + I_{AB}) (R_1 \| R_u) \frac{R_3}{(R_1 \| R_u) + R_2 + R_3} = -4V$$

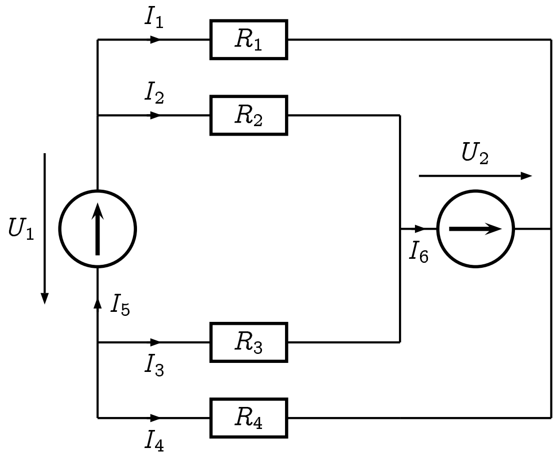
d)

$$U_L = (-I_0 + I_{AB}) (R_1 \| R_u) \frac{(R_3 \| R_L)}{(R_1 \| R_u) + R_2 + (R_3 \| R_L)} = -3.333V$$

$$P_L = U_L \cdot I_L = \frac{U_L^2}{R_L} = 0.296W$$

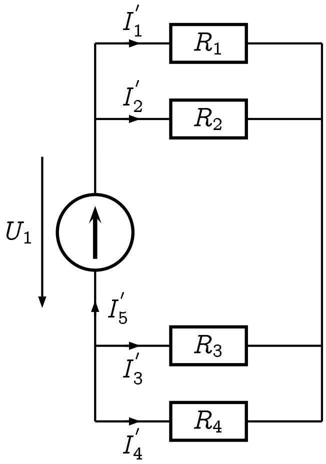
2 Übung

2.1



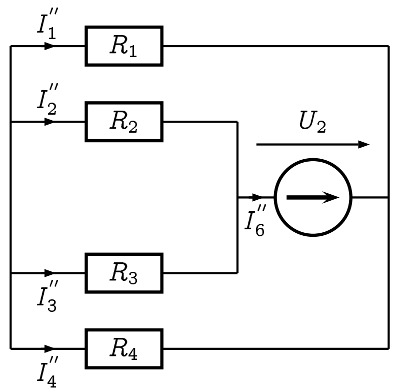
Superposition

1. $U_2 = 0$



$$
\begin{aligned}
R_{ges}' &= (R_1 \| R_2) + (R_3 \| R_4) \\
I_5' &= \frac{U_1}{R_{ges}'} = I_1' + I_2' = \frac{U_{R_1}'}{R_1} + \frac{U_{R_2}'}{R_2} \\
U_{R_1}' &= U_{R_2}' \\
I_2' &= \frac{U_{R_2}'}{R_2} = \frac{U_1}{R_{ges}'} \cdot \frac{R_1}{R_1 + R_2} \\
\\
I_5' &= \frac{U_1}{R_{ges}'} = -I_3' - I_4' = \frac{U_{R_3}'}{R_3} + \frac{U_{R_4}'}{R_4} \\
U_{R_3}' &= U_{R_4}' \\
I_3' &= \frac{U_{R_3}'}{R_3} = \frac{U_1}{R_{ges}'} \cdot \frac{R_4}{R_3 + R_4}
\end{aligned}
$$

2. $U_1 = 0$



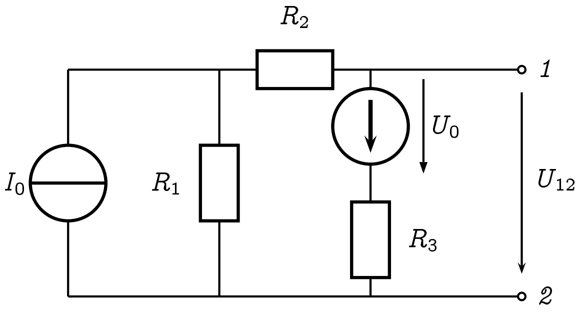
$$\begin{aligned}
R_{ges}'' &= (R_2 \| R_3) + (R_1 \| R_4) \\
I_6'' &= I_2'' + I_3'' \\
&= \frac{-U_2}{R_{ges}''} = \frac{U_{R_2}''}{R_2} + \frac{U_{R_3}''}{R_3} \\
U_{R_2}'' &= U_{R_3}'' \\
I_{R_2}'' &= \frac{U_{R_2}''}{R_2} = \frac{-U_2}{R_{ges}''} \cdot \frac{R_3}{R_2 + R_3} \\
I_{R_3}'' &= \frac{U_{R_3}''}{R_3} = \frac{-U_2}{R_{ges}''} \cdot \frac{R_2}{R_2 + R_3}
\end{aligned}$$

gegeben: $U_1 = 100V, U_2 = 150V, R_1 = 1k\Omega, R_2 = 1k\Omega, R_3 = 10k\Omega, R_4 = 10k\Omega$

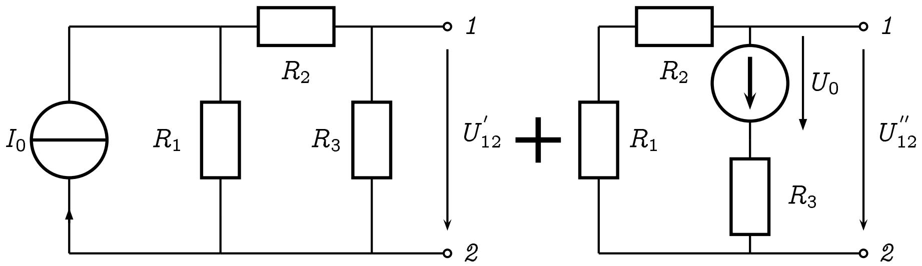
$$\boxed{\begin{aligned}
I_2 &= I_2' + I_2'' \\
&= \frac{1}{110}A - \frac{3}{40}A = -\frac{29}{440}A \\
&\approx \underline{\underline{-0,06591A}} \\
I_3 &= I_3' + I_3'' \\
&= -\frac{1}{110}A - \frac{3}{400}A = -\frac{73}{4400}A \\
&\approx \underline{\underline{-0.01659A}}
\end{aligned}}$$

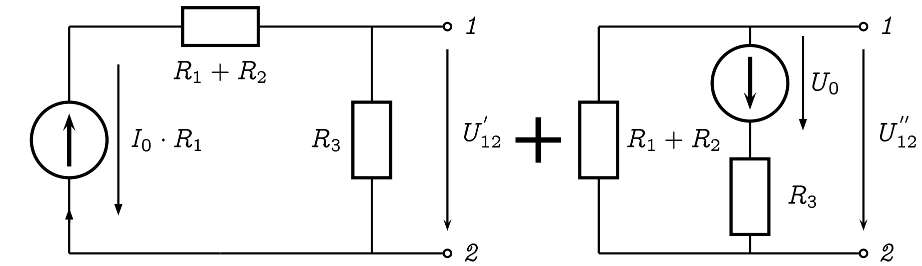
$$\begin{aligned}
I_2' &= \frac{U5\ (R4 + R3)\ R1}{R1\ R2\ R4 + R1\ R2\ R3 + R3\ R4\ R2 + R3\ R4\ R1} \\
I_2'' &= -\frac{U6\ (R4 + R1)\ R3}{R1\ R2\ R4 + R1\ R2\ R3 + R3\ R4\ R2 + R3\ R4\ R1} \\
I_2 &= \frac{U5\ R1\ R4 + U5\ R1\ R3 - U6\ R3\ R4 - U6\ R3\ R1}{R1\ R2\ R4 + R1\ R2\ R3 + R3\ R4\ R2 + R3\ R4\ R1} \\
I_3' &= -\frac{U5\ (R2 + R1)\ R4}{R1\ R2\ R4 + R1\ R2\ R3 + R3\ R4\ R2 + R3\ R4\ R1} \\
I_3'' &= -\frac{U6\ (R4 + R1)\ R2}{R1\ R2\ R4 + R1\ R2\ R3 + R3\ R4\ R2 + R3\ R4\ R1} \\
I_3 &= -\frac{U5\ R4\ R2 + U5\ R1\ R4 + U6\ R2\ R4 + U6\ R2\ R1}{R1\ R2\ R4 + R1\ R2\ R3 + R3\ R4\ R2 + R3\ R4\ R1}
\end{aligned}$$

2.2



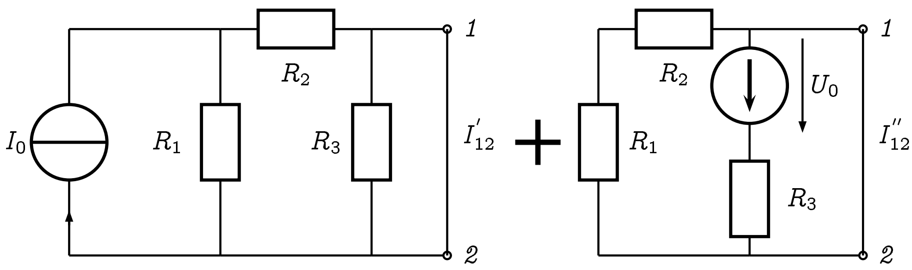
a) Leerlaufspannung U_{12}





$$
\begin{aligned}
U_{12}^{'} &= I_0 \cdot R_1 \cdot \frac{R_3}{R_1 + R_2 + R_3} \\
U_{12}^{''} &= U_1 \cdot \frac{R_1 + R_2}{R_1 + R_2 + R_3} \\
&= \\
U_{12} &= \frac{I_0 R_1 R_3 + U_1 \left(R_1 + R_2\right)}{R_1 + R_2 + R_3}
\end{aligned}
$$

b) Kurzschlussstrom I_{12}



$U_0 = 0$

Durch den Widerstand R_3 wird kein Strom fließen.

d.h. $R_1 || R_2$ und $I'_{12} = I_{R_2}$

$$I'_{12} = I_{R_1} = I_0 \cdot \frac{R_1}{R_1 + R_2}$$

$I_0 = 0$

s.o. mit $I_{R_1+R_2} = 0$

$$I''_{12} = I_{R_3} = \frac{U_0}{R_3}$$

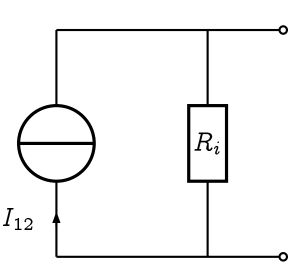
$I_{12} = I'_{12} + I''_{12}$

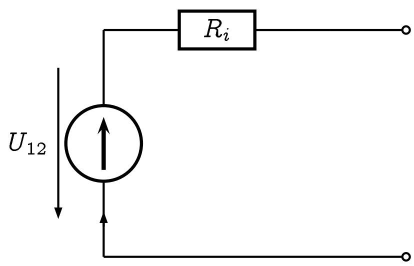
$$I_{12} = I_0 \cdot \frac{R_1}{R_1 + R_2} + \frac{U_0}{R_3}$$

c) Innenwiderstand Zweige mit Stromquellen abklemmen, Spannungsquellen kurzschließen:

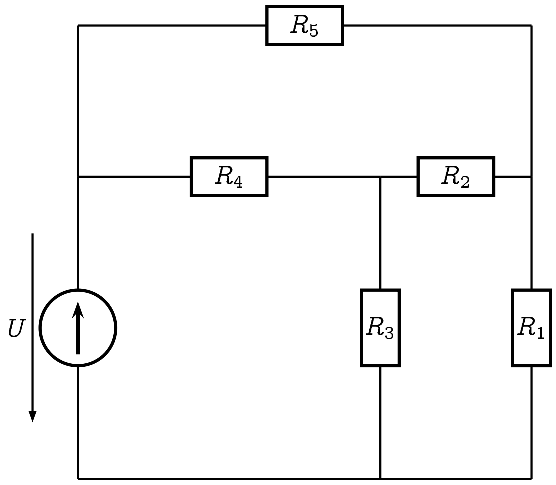
$$R_i = (R_1 + R_2) || R_3 = \frac{(R_1 + R_2) R_3}{R_1 + R_2 + R_3}$$

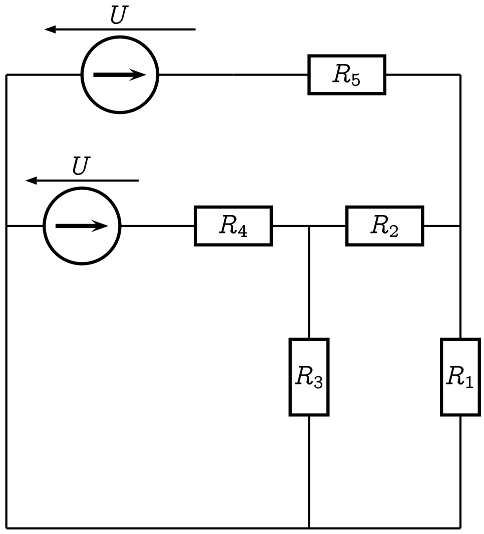
d) Ersatzstrom-, -spannungsquelle

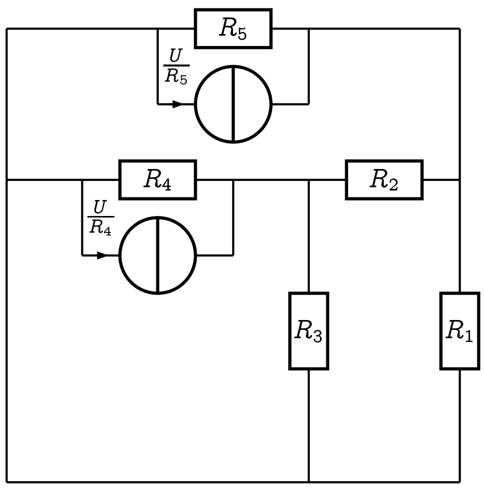




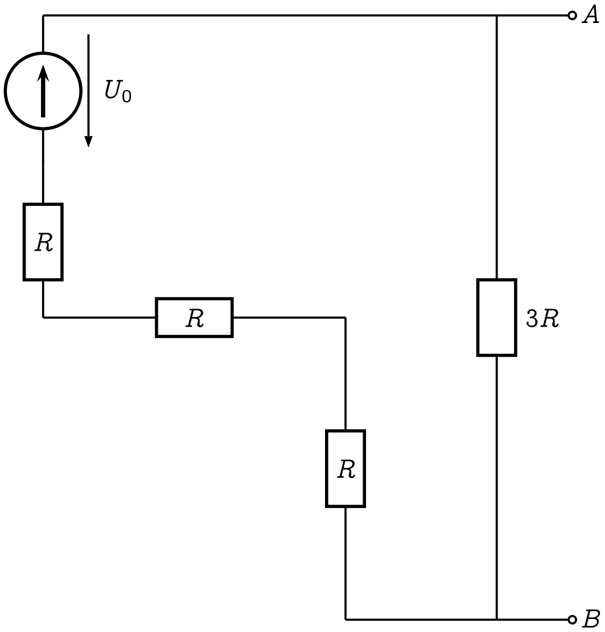
2.3







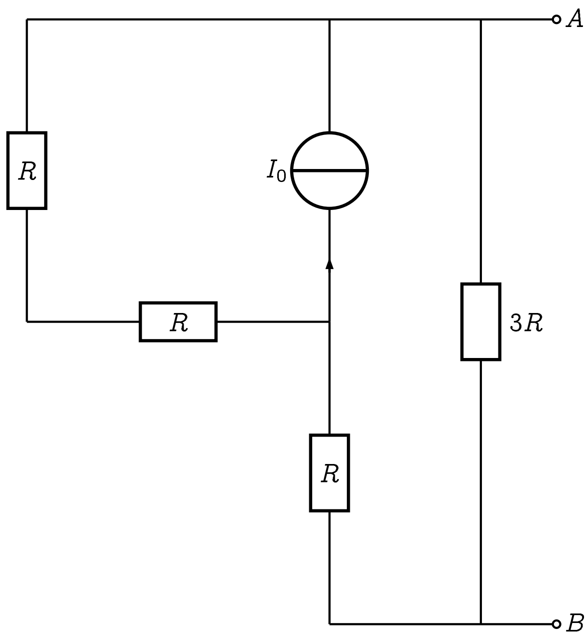
2.4



$$U'_{AB} = U_{3R}$$

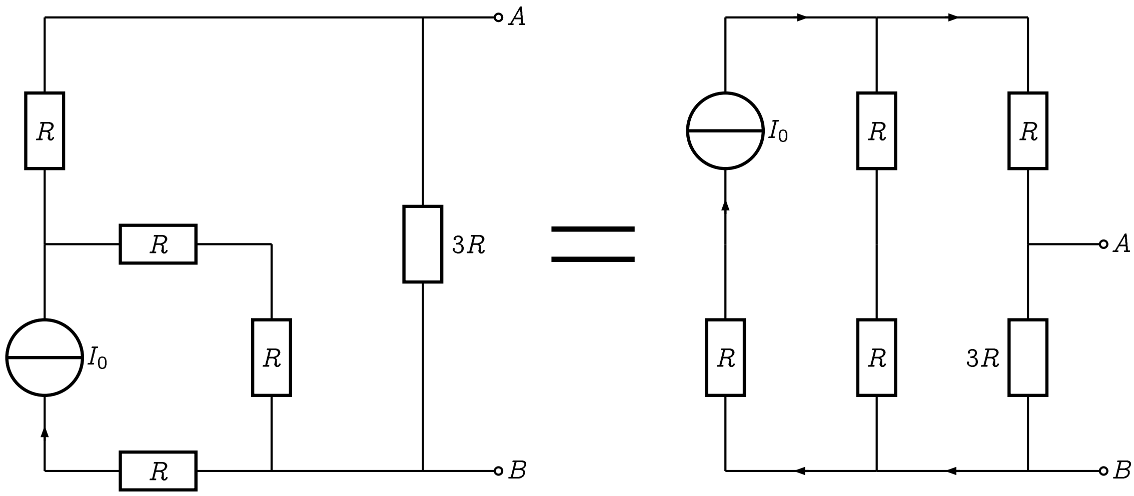
Spannungsteiler:

$$U'_{AB} = U_0 \cdot \frac{3R}{3R + (R + R + R)} = \frac{U_0}{2}$$



- die beiden Widerstände parallel zur Stromquelle zusammenfassen: $R + R = 2R$
- Stromquelle zur Spannungsquelle transformieren: $U^* = I_0 \cdot 2R$
- entstandene Reihenschaltung zusammenfassen: $2R + R = 3R$
- Spannungsteiler:

$$U''_{AB} = I_0 \cdot R$$



- Der Strom ist über einem Zweig konstant
- Stromteiler zur berechnung des Stromes im rechten Zweig:

$$I_1 = I_0 \cdot \frac{2R}{2R + 4R} = \frac{1}{3} I_0 \ R$$

- Ohmsches Gesetz: $U_{AB}^{'''} = I_1 \ 3R = \frac{1}{3} \ 3I_0 \ R = I_0 \ R$

Damit ergibt sich die Spannung der Ersatzspannungsquelle zu

$$U_{ers} = U_{AB}^{'} + U_{AB}^{''} + U_{AB}^{'''} = \frac{1}{2} U + 2 \ I_0 \ R$$

und der Ersatzwiderstand (Kurzschliessen der Spannungsquelle, Abklemmen der Stromquellen) zu

$$R_{ers} = 3R \| 3R = \frac{9R^2}{6R} = \frac{3}{2} \ R$$

2.5

Mit $P = U \cdot I$ und $I = \frac{U}{R}$ folgt $P = \frac{U^2}{R}$ Umformen ergibt: $U^2 = P \cdot R \quad \Rightarrow \quad U_{i,max} = \sqrt{P_{i,max} \cdot R_i}$

$$\begin{aligned} U_{1,max} &= \sqrt{250mW \cdot 3.6k\Omega} = 30V \\ U_{2,max} &= \sqrt{500mW \cdot 20k\Omega} = 100V \\ U_{3,max} &= \sqrt{250mW \cdot 160k\Omega} = 200V \end{aligned}$$

R_2 und R_3 sind Parallel geschaltet, d.h.

$$U_2 = U_3 = \min(U_{2,max}, U_{3,max}) = 100V$$

U_{max} folgt durch umformen der Spannungsteilergleichung (belastet mit R_3)

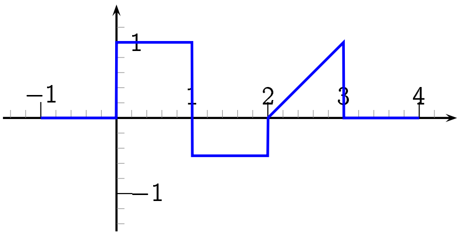
$$U_2 = U_{max} \cdot \frac{R_2 \| R_3}{R_2 \| R_3 + R_1} \quad \Rightarrow \quad U_{max} = U_2 \cdot \frac{R_2 \| R_3 + R1}{R_2 \| R_3} = 120.25V$$

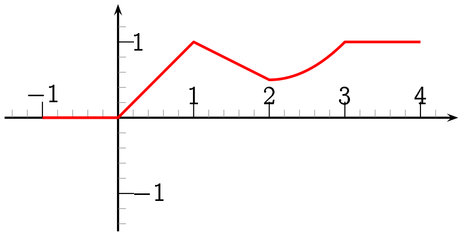
Es muss zusätzlich $U_1 \leq U_{1,max}$ gelten:

$$U_{max} = U_1 + U_2 \quad \Rightarrow \quad U_1 = U_{max} - U_2 = 20.25V \ \leq \ 30V$$

3 Übung

3.1



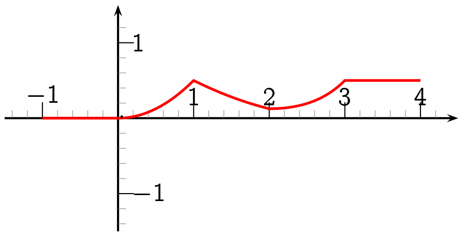


$$i(t) = \begin{cases} 0 & ,t \leq 0 \\ 1 & ,0 < t \leq 1 \\ -\frac{1}{2} & ,1 < t \leq 2 \\ t-2 & ,2 < t \leq 3 \\ 0 & ,t > 3 \end{cases}$$

$$u(t) = \frac{1}{C} \begin{cases} 0 & ,t \leq 0 \\ t & ,0 < t \leq 1 \\ -\frac{1}{2}(t-1)+1 & ,1 < t \leq 2 \\ \frac{1}{2}(t-2)^2+\frac{1}{2} & ,2 < t \leq 3 \\ 1 & ,t > 3 \end{cases}$$

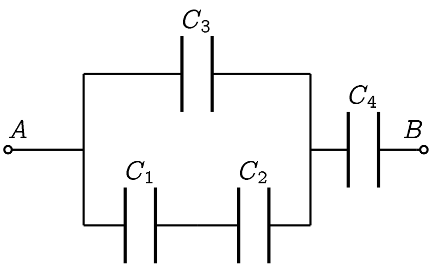
$$w(t) = \frac{1}{2}Qu(t) \quad \text{und } Q = C\ u(t)$$

$$\Rightarrow w(t) = \frac{1}{2}Cu(t)^2$$



$$w(t) = \frac{1}{2C} \begin{cases} 0 & ,t \leq 0 \\ t^2 & ,0 < t \leq 1 \\ \left(-\frac{1}{2}(t-1)+1\right)^2 & ,1 < t \leq 2 \\ \left(\frac{1}{2}(t-2)^2+\frac{1}{2}\right)^2 & ,2 < t \leq 3 \\ 1 & ,t > 3 \end{cases}$$

3.2



$$\begin{aligned} C_{AB} &= ((C_1 + C_2)\,||C_3) + C_4 \\ &= \left(\left(\left((C_1)^{-1} + (C_2)^{-1}\right)^{-1} + C_3\right)^{-1} + (C_4)^{-1}\right)^{-1} \\ &= \left(\frac{1}{\frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} + C_3} + \frac{1}{C_4}\right)^{-1} \\ &= 1.5\ \mu F \end{aligned}$$

Parallelschaltung: $\quad C_{ges} = \sum C_i$

$Q_{ges} = \sum Q_i$

Reihenschaltung: $\frac{1}{C_{ges}} = \sum \frac{1}{C_i}$
$Q_{ges} = Q_i = Q_1 = Q_2 = \ \ldots$

$U_{AB} = 15V$ für $t \to \infty$, mit $Q_{AB} = C_{AB}\ U_{AB}$ und $Q_4 = C_4\ U_4$ folgt

$$Q_{AB} = Q_4 \qquad U_4 = \frac{C_{AB}}{C_4}\ U_{AB} = \frac{1.5\ \mu F}{2\ \mu F} \cdot 15V = 11.25V$$

$U_{AB} = U_{123} + U_4$ ergibt $U_{123} = 15V - 11.25V = 3.75V$

Parallelschaltung aus C_{12} und C_3: $U_{123} = U_{12} = U_3 = 3.75V$
Reihenschaltung aus C_1 und C_2: $Q_{12} = Q_1 = Q_2 = U_{123}\ C_{12}$

$$U_{123}\ \frac{C_{12}}{C_2} = U_2 = 2.5V$$

$$U_1 = U_{123} - U_2 = 3.75V - 2.5V = 1.25V$$

3.3

a) Platte P ist mit Elektrode 2 elektrisch leitend verbunden

$$C_{1P} = \frac{\epsilon_L\ A}{x}$$

b) Platte P ist mit Elektrode 1 elektrisch leitend verbunden

$$C_{PK} = \frac{\epsilon_0 \epsilon_K\ A}{b} \qquad C_{K2} = \frac{\epsilon_0 \epsilon_L\ A}{d - x - a - b}$$

$$C_{P2} = \frac{C_{PK} \cdot C_{K2}}{C_{PK} + C_{K2}} = \frac{\epsilon_0 \epsilon_L \epsilon_K A}{\epsilon_K\,(d - x - a - b) + \epsilon_L b}$$

c) Platte P ist elektrisch isoliert

$$C_{12} = \frac{C_{1P} \cdot C_{P2}}{C_{1P} + C_{P2}}$$

3.4

a)

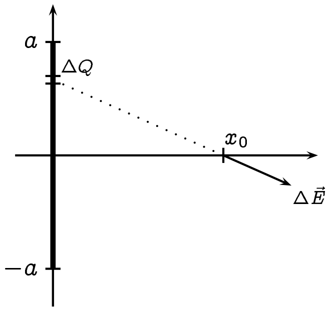
$$\vec{E} = \vec{E_1} + \vec{E_2} = \frac{-Q}{4\pi\epsilon\left(\frac{a}{2} - x\right)^2} \cdot (-\vec{e_x}) + \frac{Q}{4\pi\epsilon\left(\frac{a}{2} + x\right)^2} \cdot (\vec{e_x}) \qquad \text{,für } -\frac{a}{2} + r_0 < x < \frac{a}{2} - r_0$$

b)

$$\vec{E} = \vec{E_1} + \vec{E_2} = \frac{Q}{4\pi\epsilon\left(\frac{a}{2} - x\right)^2} \cdot (-\vec{e_x}) + \frac{Q}{4\pi\epsilon\left(\frac{a}{2} + x\right)^2} \cdot (\vec{e_x}) \qquad \text{,für } -\frac{a}{2} + r_0 < x < \frac{a}{2} - r_0$$

3.5

a)



b)

$$|\Delta \vec{E}| = \frac{\Delta Q}{4\pi\epsilon r^2} \qquad \text{mit } r = \sqrt{x_0^2 + y^2} \quad \rightarrow \; |\Delta \vec{E}| = \frac{\Delta Q}{4\pi\epsilon\left(x_0^2 + y^2\right)}, \; -a \leq y \leq a$$

c) aus MuLö:
"Zu jeder Teilladung auf der positiven y-Achse gibt es eine Teilladung auf der negativen y-Achse, so dass sich die y-Komponenten der Feldstärken gerade Aufheben (Symmetrie). $\Rightarrow \quad \vec{E} = (\vec{e_x})$"

d) *siehe ElCapitan*

3.6

Hinweis:

1. die Hüllflächen umschließen jeweils Platte a, bzw b (Ober- und Unterseite = 2A)
2. Ladung gleichmäßig verteilt $\quad \rightarrow \vec{D} = const(A)$
3. Richtung von $\vec{D}$ bzw $\vec{E}$ ist:
   - oberhalb jeder Platte: $+\vec{e}_y$
   - unterhalb jeder Platte: $-\vec{e}_y$

a)

$$Q = \oint_A \vec{D} \cdot d\vec{A} \quad \rightarrow \quad Q = D \cdot A \quad \Rightarrow \; D = \frac{Q}{A}$$

$$\vec{D}_a = \frac{3Q}{2A} \qquad\qquad \vec{D}_B = \frac{2Q}{2A}$$

	$\vec{D_a}$	$\vec{D_b}$	$\vec{D} = \vec{D_a} + \vec{D_b}$
$y < -\frac{d}{2}$	$\frac{3Q}{2A} \cdot (-\vec{e_y})$	$\frac{Q}{A} \cdot (-\vec{e_y})$	$\frac{5Q}{2A} \cdot (-\vec{e_y})$
$-\frac{d}{2} < y < \frac{d}{2}$	$\frac{3Q}{2A} \cdot (-\vec{e_y})$	$\frac{Q}{A} \cdot (+\vec{e_y})$	$\frac{Q}{2A} \cdot (-\vec{e_y})$
$y > \frac{d}{2}$	$\frac{3Q}{2A} \cdot (+\vec{e_y})$	$\frac{Q}{A} \cdot (+\vec{e_y})$	$\frac{5Q}{2A} \cdot (+\vec{e_y})$

b)

$$U_{ab} = E \cdot d \qquad \vec{D} = \vec{E} \cdot \epsilon \qquad \rightarrow \quad U_{AB} = \frac{Q\ d}{2\ A\ \epsilon}$$

c)

$$\vec{F}_a = Q_a\ \vec{E}_b = Q_a \frac{\vec{D}_b}{\epsilon} = \frac{3Q\ 2Q}{2A\ \epsilon} = \frac{3\ Q^2}{A\ \epsilon}$$
$$\vec{F}_b = Q_b\ \vec{E}_a = Q_b \frac{\vec{D}_a}{\epsilon} = \frac{2Q\ 3Q}{2A\ \epsilon} = \frac{3\ Q^2}{A\ \epsilon}$$

4 Übung

4.1

$$F = I\ l\ B\ = I\ l\ \mu\ H = \frac{I^2\ l\ \mu_0\mu_r}{2\ \pi\ d}$$
$$= \frac{100^2 \cdot 2 \cdot 1 \cdot 4 \cdot \pi}{2 \cdot \pi \cdot 0.1 \cdot 10^7}\ N = 0.04\ N$$

$$[N] = \frac{[AVs]}{[m]} \qquad \text{mit } [AV] = [P] = \frac{[Nm]}{[s]}$$

4.2

a)

$$\Theta = i(t) \cdot N_1 = \frac{\Phi}{\Lambda_{Fe}} + \frac{\Phi}{\Lambda_L}$$
$$= \frac{\Phi}{A}\left(\frac{l_{Fe}}{\mu_0\mu_{Fe}} + \frac{l_L}{\mu_0}\right)$$

$$\Phi(t) = \frac{i(t) \cdot N_1\ A}{\frac{l_{Fe}}{\mu_0\mu_{Fe}} + \frac{l_L}{\mu_0}} \quad \left[\frac{A\ m^2}{\frac{m}{\frac{Vs}{Am}} + \frac{m}{\frac{Vs}{Am}}}\right]$$
$$= \frac{0.1 \cdot 2000 \cdot 4 \cdot 10^{-4}}{\frac{0.2\ \pi}{3000 \cdot 4\pi \cdot 10^{-7}} + \frac{1 \cdot 10^{-3}}{4\pi \cdot 10^{-7}}} \cdot \sin\left(31.42\frac{1}{s} \cdot t\right) Vs$$
$$= 83.122 \cdot 10^{-6} \cdot \sin\left(31.42\frac{1}{s} \cdot t\right) Vs$$

b)

$$\Phi(t) = \int B dA$$
$$= BA$$
$$B = \frac{\Phi(t)}{A} = \frac{83.122}{4} \cdot 10^{-2} \cdot \sin\left(31.42\frac{1}{s} \cdot t\right) \frac{Vs}{m^2}$$
$$\hat{B} = 0.208\ T$$

c)

$$u_2(t) = N_2 \cdot \frac{d}{dt}\Phi(t)$$
$$= 5000 \cdot 83.122 \cdot 10^{-6} \cdot 31.42 \cos\left(31.42\frac{1}{s} \cdot t\right)\ V$$
$$= 13.06 \cdot \cos\left(31.42\frac{1}{s} \cdot t\right)\ V$$

4.3

- Querschnittsfläche A ist quadratisch: $A = a^2 = 4cm^2 \quad \Rightarrow a = 2\ cm$
- mittlere Länge $l_{m,Fe} = 10cm + 10cm + 8cm + 7.5cm = 35.5cm \qquad l_{m,L} = 0.5cm$
- $A = A_L = A_{Fe} \quad \rightarrow \quad B = B_L = B_{Fe}$ und H_{Fe} aus Tabelle

a)

$$
\begin{aligned}
B &= 1T \quad \rightarrow \quad H_{Fe} = 3\frac{A}{cm} \\
\Theta &= H_{Fe} \cdot l_{m,Fe} + H_L \cdot l_{m,L} \\
&= H_{Fe} \cdot l_{m,Fe} + \frac{B_L}{\mu_0} \cdot l_{m,L} \\
&= 3\frac{A}{cm} \cdot 35.5\ cm \ + \ \frac{1}{4\ \pi\ 10^{-7}}\ \frac{A}{m} \cdot 0.005\ m \\
\Theta &= 4085.37\ A
\end{aligned}
$$

b)

$$
\frac{\Theta_L}{\Theta} = \frac{3978.87}{4085.37} = 0.974 \quad \rightarrow \ 97.4\%
$$

c)

$$
\begin{aligned}
B &= 1.9T \quad \rightarrow \quad H_{Fe} = 200\frac{A}{cm} \\
\Theta &= 200 \cdot 35.5\ A + \frac{1.9}{\pi}\ 10^7 \cdot 0.005\ A \\
&= 14659.86A
\end{aligned}
$$

d)

$$
\frac{\Theta_L}{\Theta} = \frac{7559.86}{14659.86} = 0.516 \quad \rightarrow \ 51.6\%
$$

4.4

coming soon ...

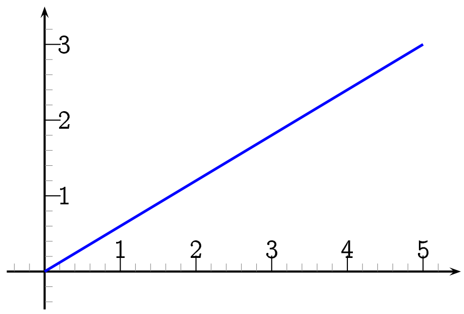
4.5

a)

Annahme: $i_1(t)$ sei linear $\Rightarrow$

$$
i_1(t) = \frac{3\ A}{5\ s} \cdot t
$$

(kann ich dem text nicht entnehmen)



$$
\begin{aligned}
U &= -n_2 \frac{d\Phi}{dt} \qquad \textbf{mit } \Phi = \int B\ dA = B(t)\ A_2 \\
&= -n_2 \frac{d}{dt} B(t)\ A_2 \\
B(t) &= i_1(t) \cdot \frac{\mu_0\ n_1}{l_1} \\
\Phi(t) &= i_1(t) \cdot \frac{\mu_0\ n_1\ A_2}{l_1} \\
\rightarrow U &= -\frac{d}{dt} i(t) \cdot \frac{\mu_0\ n_1\ n_2\ A_2}{l_1} \\
&= -\frac{3\ A}{5\ s} \cdot \frac{4\pi \cdot 3500 \cdot 80 \cdot \pi \cdot 0.02^2}{1 \cdot 4 \cdot 10^7} \frac{V\ s\ m^2}{A\ m\ m} \\
&= -6.63 \cdot 10^{-5}\ V
\end{aligned}
$$

b) Drehung der Feld- oder Induktionsspule ... *siehe MuLö*

4.6

Geschwindigkeitsvektor des Elektrons

$$\vec{v} = (v_x, v_y, v_z)^T$$

Elektron fliegt geradlinig

$$\rightarrow\ v_y = v_z = 0 \quad \Rightarrow \quad |\vec{v}| = v_x = v$$

Kräfte, die auf das Elektron wirklen:

$$\vec{F} = \vec{F}_m + \vec{F}_e$$

Kraft infolge Magnetfeld:

$$\vec{F}_m = q \cdot \left(\vec{v} \times \vec{B}\right) = q_e \cdot v \cdot B \cdot \vec{e}_z \qquad \text{,mit } \vec{B} = (0, B, 0)^T$$

Kraft infolge E-Feld :

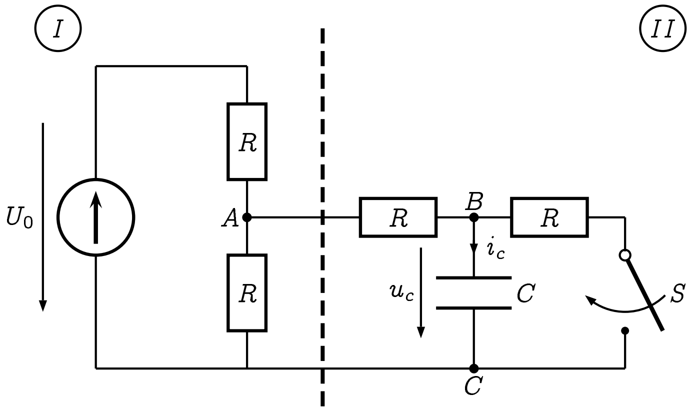
$$\vec{F}_e = q_e \cdot \vec{E} = q_e \cdot \frac{U}{d} \cdot (-\vec{e}_z)$$

die Richtungen von $\vec{F}_e$ und $\vec{F}_m$ sind genau entgegengesetzt in z-Richtung!
keine Bewegungsänderung ohne äußere Krafteinwirkung: $\vec{F}_e + \vec{F}_m = 0$

$$|\vec{F}_e| \stackrel{!}{=} |\vec{F}_m| \qquad \rightarrow\ \frac{U}{d} = v \cdot B \qquad \rightarrow\ v = \frac{U}{d \cdot B}$$

5 Übung

5.1



a) Da S offen ist, fließt duch II kein Strom. Die Spannung an diesen Widerständen ist 0. Für den Innenwiderstand muss R_{AB} berücksichtigt werden.

$$U_{ers} = U_{AC} = U_0 \cdot \frac{R}{R+R} = \frac{U_0}{2} = 25\ V$$
$$R_i = R + (R \| R) = \frac{3}{2} R = 1.5\ k\Omega$$

b) S ist geschlossen.

$$U_{ers} = U_{BC} = U_{AC} \cdot \frac{R}{R+R}$$
$$U_{AC} = U_0 \frac{(R\|2R)}{R + (R\|2R)} = \frac{2}{5} U_0$$
$$\rightarrow U_{ers} = \frac{2}{5} U_0 \frac{R}{2\ R} = \frac{1}{5} U_0 = 10\ V$$

$$R_i = (R + (R\|R))\, \| R = \frac{3}{5} R = 0.6\ k\Omega$$

c)

$$\begin{pmatrix} 2R & R \\ R & 2R \end{pmatrix} \begin{pmatrix} I_0 \\ i_C \end{pmatrix} = \begin{pmatrix} -U_0 \\ -u_C \end{pmatrix}$$

$$u_c = \frac{1}{5} U_0$$
$$\rightarrow i_c = \frac{U_0 - 2\ u_C}{3\ R} = \frac{1}{100} A = 10\ mA$$

d)

$$\begin{pmatrix} 2R & R & R \\ R & 2R & 2R \\ R & 2R & 3R \end{pmatrix} \begin{pmatrix} I_0 \\ i_c \\ i_? \end{pmatrix} = \begin{pmatrix} -U_0 \\ -u_c \\ 0 \end{pmatrix}$$

$$u_c = \frac{1}{2} U_0$$
$$\rightarrow i_c = \frac{U_0 - 5\ u_C}{3\ R} = \frac{-1}{40} A = -25\ mA$$

5.2

$$u_L = L \frac{d}{dt} i_L$$
$$U = i_L \cdot R_1 + u_L = i_L \cdot R_1 + L \frac{d}{dt} i_L$$
$$\frac{U}{L} = \frac{R_1}{L} i_L + \frac{d}{dt} i_L$$

homogene Lösung:

$$\frac{d}{dt} i_L + \frac{R_1}{L} i_L = 0$$
$$\frac{d}{dt} i_L = -\frac{R_1}{L} i_L$$
$$\frac{di}{i_L} = -\frac{R_1}{L}\ dt$$
$$\ln |i_L| = -\frac{R_1}{L} t + \ln |k|$$
$$i_L = k \cdot \mathrm{e}^{-\frac{R_1}{L} t}$$

partikuläre Lösung:

$$i_L = A \qquad \rightarrow \frac{d}{dt} i_L = 0$$
$$A \cdot \frac{R_1}{L} = \frac{U}{L}$$
$$A = \frac{U}{R_1}$$

Anfangswertproblem:

$$\rightarrow i_L = k \cdot \mathrm{e}^{-\frac{R_1}{L} t} + \frac{U}{R_1}$$
$$i_L(0) = 0 \qquad 0 = k \cdot \mathrm{e}^{-\frac{R_1}{L} t} + \frac{U}{R_1}$$
$$k = -\frac{U}{R_1}$$

$$\Rightarrow i_L = \frac{U}{R_1} \left(1 - \mathrm{e}^{-\frac{R_1}{L} t}\right)$$